

A Full 24-Parameter MSSM Exploration

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Abstract. Up until now a complete scan in all phenomenologically relevant directions of the MSSM at the TeV scale for performing global fit has not been done. Given the imminent start of operation of the LHC, this is a major gap on our quest to discovering and understanding the physical implications of low energy supersymmetry. The main reason for this is the large number of parameters involved that makes it computationally extremely expensive using the traditional methods. In this talk I demonstrate that with advanced sampling techniques the problem is solvable. The results from the explored 24-parameter TeV scale MSSM (phenoMSSM) are remarkably distinct from previous studies and are independent of models for supersymmetry breaking and mediation mechanisms. Hence they are a more robust guide to searches for supersymmetry and dark matter.

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PhenoMSSM

The 105 free physical parameters of the MSSM with R-parity [1] makes a complete study of supersymmetry an impossible task. For practical purposes phenomenologists had to construct models with fewer parameters at unification scale from which RGEs were used to obtain the lower-energy (M_{susy}) scale sparticle spectrum and properties. Most famous among this class of constructions is mSUGRA/CMSSM which has just 4 parameters and a \pm sign. This has been used for providing benchmark points for sparticle searches and phenomenology. But the approach has important limitations. Supersymmetry discovery definitely requires probe of large regions, in a maximal manner, of its parameter space. Moreover, it may be misleading if the models used to interpret experimental results are not realistic or not the most general. Hence the study of MSSM in its complete parameters around the electroweak scale is a more natural approach. This will be independent of supersymmetry breaking models, hidden-sector physics, mediation mechanisms and renormalisation group running interpolations.

In the following sections I demonstrate this more natural approach by applying advanced Monte Carlo technique, called nested sampling [2] which is implemented in the currently private code MultiNest [3], to fully explore the MSSM with R-parity and minimal flavour violation (MFV) [4] in its entire, 24-dimensional, most phenomenologically important parameters at the scale $M_{\text{susy}} \sim 1$ TeV – a set-up we call **phenoMSSM** [5]. The codes SFitter and Fittino [6] can reconstruct (weak-scale) MSSM parameters from collider data but the main goal here is to explore all the parameters, perform a global fit to current indirect data and draw inferences for LHC and (future) LC physics. I start with defi-

nition of Bayes' theorem, describe how its variables were constructed for the **phenoMSSM** and then give sample results from the exercise of applying the theorem on the model before concluding in the last section.

Bayesian Inference in Particle Physics:. Bayes' theorem is at the core of the algorithm used to efficiently explore the entire viable parameter regions of the model. It states that

$$P(\theta|D, H) = P(D|\theta, H)P(\theta|H)/P(D|H) \quad (1)$$

where $P(\theta|H)$ is the parameters prior density distribution representing the conditional probability of a set of parameters θ given that the model or hypothesis, H , is true. It states what values the model parameters are expected to take. With some set of predictions (data or observables), D , obtained from the model, $P(D|\theta, H)$ quantifies the likelihood for the model, at the given parameter values, to be true. For a model with n parameters the n -dimensional integral $Z = P(D|H) = \int P(D|\theta, H)P(\theta|H)d\theta$ represents the evidence for the model. $P(\theta|D, H)$ is the posterior probability density function and gives a measure of how well the set of parameters predicts the given data set, D . Nested sampling is a general method for evaluating this n -dimensional integral by converting it to a 1-dimensional integral over a unit interval [2]. The sampling procedure also produces the posterior probability distribution eqn. (1) as by-product.

PhenoMSSM parameters, θ : The soft supersymmetry breaking part of the MSSM Lagrangian density have contributions from different types of interactions, $\mathcal{L}_{\text{soft}} = \mathcal{L}_{\text{gauginos}} + \mathcal{L}_{\text{sfermions}} + \mathcal{L}_{\text{trilinear}} + \mathcal{L}_{\text{higgs}}$, and sources the 105 supersymmetric free parameters. In order to suppress CP-violation and FCNC real soft

terms, diagonal sfermions masses and trilinear couplings, and 1st/2nd generation squark masses and slepton masses degeneracies were assumed. A_t , A_b and A_τ are the most important trilinear couplings but we also include $A_e = A_\mu$ because it is relevant for $(g-2)_\mu$ computation [7]. All the other trilinear couplings and neutrino masses are set to zero. This way the total number of free parameters becomes 20. Adding to these the 4 most important SM “nuisance” parameters: $\{m_t, m_b(m_b)^{\overline{MS}}, \alpha_{em}(m_Z)^{\overline{MS}}, \alpha_s(m_Z)^{\overline{MS}}\}$, makes a total of 24 physical parameters in **phenomMSSM**. These are listed and described in Table 1. Representing them in a 24-

TABLE 1. The 24 parameters of **phenomMSSM** set-up.

Parameter	Description
M_1, M_2, M_3	Bino, Wino and Gluino masses
$m_{\tilde{e}_L} = m_{\tilde{\mu}_L}$	1st/2nd generation L_L slepton masses
$m_{\tilde{\tau}_L}$	3rd generation L_L slepton mass
$m_{\tilde{e}_R} = m_{\tilde{\mu}_R}$	1st/2nd generation E_R sleptons masses
$m_{\tilde{\tau}_R}$	3rd generation E_R slepton mass
$m_{\tilde{u}_L} = m_{\tilde{d}_L}$	
$m_{\tilde{c}_L} = m_{\tilde{s}_L}$	1st/2nd generation Q_L squark masses
$m_{\tilde{t}_L} = m_{\tilde{b}_L}$	3rd generation Q_L squark masses
$m_{\tilde{u}_R} = m_{\tilde{c}_R}$	1st/2nd generation U_R squark masses
$m_{\tilde{t}_R}$	3rd generation U_R squark mass
$m_{\tilde{d}_R} = m_{\tilde{s}_R}$	1st/2nd generation D_R squark masses
$m_{\tilde{b}_R}$	3rd generation D_R squark mass
$A_{t,b,\tau}$	top, b- and τ - quark trilinear couplings
$A_e = A_\mu$	μ and e trilinear couplings
$m_{H_{1,2}}$	up- and down-type Higgs doublet masses
$\tan\beta$	scalar doublets $\mathbf{v}_{\mathbf{v}}$ ratio
m_t	top quark pole mass
$m_b(m_b)^{\overline{MS}}$	b-quark mass
$1/\alpha_{em}(m_Z)^{\overline{MS}}$	electromagnetic coupling constant
$\alpha_s(m_Z)^{\overline{MS}}$	strong coupling constant

dimensional vector θ , the combined prior for the model is

$$\pi(\theta) = P(\theta|H) = \pi(\theta_1)\pi(\theta_2)\dots\pi(\theta_{24}). \quad (2)$$

Set of Observables, D : We use high precision electroweak and B-physics collider observables and the dark matter relic density from WMAP5 results (all shown in Table 2) to study the viable parameter regions of the **phenomMSSM**. Its predictions for these observables were obtained from the 24 input parameters via **SOFTSUSY2.0.17** [12] for producing the MSSM spectrum; **micrOMEGAS2.1** [13] for computing neutralino dark matter relic density, the branching ratio $BR(B_s \rightarrow \mu^+ \mu^-)$ and the anomalous magnetic moment of the muon $(g-2)_\mu$; **SuperIso2.0** [14] for predicting the Isospin asymmetry in the decays $B \rightarrow K^* \gamma$ and $BR(b \rightarrow s \gamma)$ with all NLO supersymmetric QCD and NNLO SM QCD contributions included; and **susyPOPE** [15] for computing W -boson mass m_W , the effective leptonic mixing angle variable $\sin^2 \theta_{eff}^{lep}$, and the

total Z-boson decay width, Γ_Z , at two loops in the dominant MSSM parameters. These physical observables de-

TABLE 2. A summary of the 11 observables. $\Omega_{CDM}h^2$ error is inflated to 0.02 to accommodate theoretical uncertainties.

Observable	Mean value	Uncertainty
m_W	80.398 GeV	0.0025 GeV
Γ_Z	2.4952 GeV	0.0023 GeV
$\sin^2 \theta_{eff}^{lep}$	0.23149	0.000173
$\delta a_\mu \times 10^{10}$	29.5	8.8
$Br(b \rightarrow s \gamma) \times 10^4$	3.55	0.72
m_h	114.4 GeV	lower limit
$Br(B \rightarrow \mu^+ \mu^-)$	5.8×10^{-8}	upper limit
$R_{\Delta M_{B_s}}$	0.85	0.11
$R_{Br(B_u \rightarrow \tau \nu)}$	1.2589	0.4758
Δ_{0-}	0.0375	0.0289
$\Omega_{CDM}h^2$	0.1143	0.02

rived from the model parameters form the data set, D . For each element, D_i , in D the likelihood $L(\theta) = P(D_i|\theta, H)$ was calculated. Assuming that the observables are independent then the combined likelihood for the model is

$$L(\theta) = \prod_i (2\pi\sigma_i^2)^{-1/2} \exp[-(O_i - \mu_i)^2/2\sigma_i^2] \quad (3)$$

where O_i is the predicted value of the i th observable with $i = 1, 2, 3, \dots, 24$ and σ_i its corresponding standard error.

With all the above Bayesian inference parameters set and ready the **Multinest** code was employed for (guided) sampling of the 24 parameters. At each parameter-space point the values were passed in SUSY Le Houches Accord (SLHA) format [8] to the different particle physics software used for predicting the physical observables. The predictions were then checked against experimental values with (non)deviations quantified by the likelihood function. Next, the likelihoods modulate the parameter prior probabilities to produce the Bayesian evidence and posterior probability distributions for the model. Two prior probability density ranges, 1 TeV and 2 TeV, were used for the purely supersymmetric parameters (the first 20 listed in Table 1) with the gaugino masses and trilinear couplings allowed to take both positive and negative values. The slepton and squark masses were bounded from below at 100 GeV. The SM parameters were taken as Gaussian noise around their mean experimental values: $m_t = 172.6 \pm 1.4$, $m_b(m_b)^{\overline{MS}} = 4.2 \pm 0.07$, $1/\alpha_{em}(m_Z)^{\overline{MS}} = 127.918 \pm 0.018$ and $\alpha_s(m_Z)^{\overline{MS}} = 0.1172 \pm 0.002$.

Results. Here I give some results from the exploration exercise, more results and analysis on the global fit to data is in progress [5]. The results are quite robust under change of parameter prior ranges. Figure 1 shows that m_{h^0} is most likely around 115 to 117 GeV,

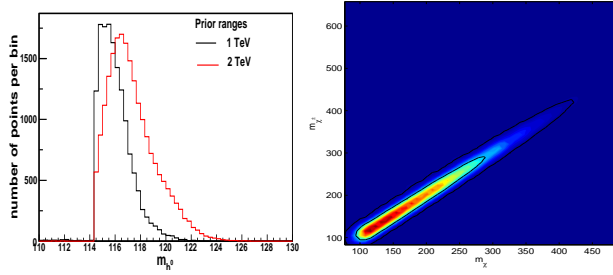


FIGURE 1. Left: Posterior probability distribution (ppd) for m_{h^0} . Right: Marginalised ppd showing chargino co-annihilation; colour coded: 0 for blue to red/black for maximum.

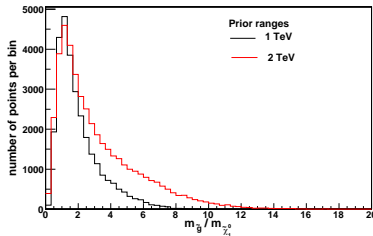


FIGURE 2. ppd of gluino to neutralino mass ratio showing most probable phenoMSSM values around 2 and severely dis-favouring mSUGRA/CMSSM and AMSB.

just above the LEP limit. The different neutralino dark matter (DM) annihilation mechanisms at early times of the universe that leads to the value of its relic density today are important in defining different phenomenological regions in model parameter space. A profound feature of the **phenoMSSM** is that it shows lots of chargino co-annihilation as shown in Figure 1. The gluino to neutralino mass ratio is an interesting quantity that characterises different models of supersymmetry (breaking.) For instance mSUGRA(AMSB) with predominantly bino(wino) LSP has $m_{\tilde{g}}/m_{\tilde{\chi}_1^0} \approx 6(9)$ [16]. The mirage mediation [17] and the LARGE volume [18] scenarios have the characteristic ratio less than 6 and between 3 to 4 respectively. Figure 2 shows the ppd of this quantity, providing discrimination between the **phenoMSSM** and the other models. This feature is robust to prior parameter ranges.

Conclusion:. With advanced Bayesian technique it is possible to fully explore weak-scale MSSM in all of its phenomenologically relevant parameter directions. Doing this is very important since low energy supersymmetry is the main focus of the LHC supersymmetry-search experiments. Hence such low energy approach as demonstrated here will be a better guide and much more realistic. The **phenoMSSM** has a characteristic $m_{\tilde{g}}/m_{\tilde{\chi}_1^0} \lesssim 2$

and shows the existence of chargino-neutralino (DM) co-annihilation. The whole procedure can be applied to other BSM constructions such as those with additional CP-violating and FCNC sources and non-zero neutrino masses. Moreover, using the evidence value, Z , the technique is a powerful tool for model comparisons [19].

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REFERENCES

1. P. Fayet, *Phys. Lett. B* **64** (1976) 159; **69** (1977) 489; "Unification of the Fundamental Particle Interactions," Proc. Erice Conf. (Plenum, N. Y., 1980) p.587; S. Dimopoulos and H. Georgi, "Supersymmetric GUTs", Second Workshop on Grand Unification, University of Michigan, Ann Arbor, April 24-26, 1981, eds. J. Leveille, L. Sulak and D. Unger (Birkhauser, 1981)p. 285; S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193** (1981)150.
2. J. Skilling, AIP Conference Proceedings of the 24th International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering, Vol. 735, pp. 395-405 (2004)
3. F. Feroz and M. P. Hobson, arXiv:0704.3704 [astro-ph].
4. A. J. Buras, P. Gambino, M. Gorbahn, S. Jäger, and L. Silvestrini, *Phys. Lett. B* **500** 161 (2001).
5. S.S. AbdusSalam, B.C. Allanach, F. Quevedo, F. Feroz and M. P. Hobson, to be released.
6. R. Lafaye, T. Plehn and D. Zerwas, arXiv:hep-ph/0404282; P. Bechtle, K. Desch and P. Wienemann, *Comput. Phys. Commun.* **174**, 47 (2006).
7. S. P. Martin and J. D. Wells, *Phys. Rev. D* **64** (2001) 035003 [arXiv:hep-ph/0103067].
8. P. Skands *et al.*, *JHEP* **0407** (2004) 036.
9. J. Alcaraz *et al.* [ALEPH Collaboration], arXiv:hep-ex/0612034.
10. M. J. Ramsey-Musolf and S. Su, arXiv:hep-ph/0612057.
11. J. R. Ellis, S. Heinemeyer, K. A. Olive, A. M. Weber and G. Weiglein, *JHEP* **0708** (2007) 083 [arXiv:0706.0652].
12. B. C. Allanach, *Comput. Phys. Commun.* **143** (2002) 305.
13. G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, arXiv:0803.2360 [hep-ph] and references therein.
14. F. Mahmoudi, arXiv:0710.2067 [hep-ph].
15. A. M. Weber *et al.*, forthcoming; S. Heinemeyer, W. Hollik, D. Stockinger, A. M. Weber and G. Weiglein, *JHEP* **0608** (2006) 052 [arXiv:hep-ph/0604147].
16. See for instance H. P. Nilles talk at SUSY08.
17. O. Lebedev, H. P. Nilles and M. Ratz, arXiv:hep-ph/0511320.
18. J. P. Conlon, S. S. Abdussalam, F. Quevedo and K. Suruliz, *JHEP* **0701**, 032 (2007); J. P. Conlon, C. H. Kom, K. Suruliz, B. C. Allanach and F. Quevedo, *JHEP* **0708**, 061 (2007).
19. F. Feroz, B. C. Allanach, M. Hobson, S. S. AbdusSalam, R. Trotta and A. M. Weber, arXiv:0807.4512 [hep-ph].